Principles Regarding the Common Core State Standards for Mathematics

Focus andCoherence

The two major evidence-based principles on which the standards are based are ***focus*** and ***coherence.*** **Focus** is necessary so that students have sufficient time to think, practice, and integrate new ideas into their growing knowledge structure. Focus is also a way to allow time for the kinds of rich classroom discussion and interaction that support the Standards for Mathematical Practice.

The second principle, **coherence,** arises from mathematical connections. Some of the connections in the standards knit topics together at a single grade level (such as area models and multiplication in grade 3). Most connections, however, play out across two or more grade levels to form a progression of increasing knowledge, skill, or sophistication. The standards are woven of these progressions. Likewise, instruction at any given grade would benefit from being informed by a sense of the overall progression students are following across the grades.

Another set of connections is found between the content standards and the practice standards. These connections are absolutely essential to support the development of students? broader mathematical understanding. To reflect the standards, the Model Content Frameworks emphasize that mathematics is not a checklist of fragments to be mastered, but that doing and using mathematics involves connecting content and practices.

Focus is critical to ensure that students learn the most important content completely, rather than succumb to an overly broad survey of content. Coherence is critical to ensure that students see mathematics as a logically progressing discipline, which has intricate connections among its various domains and requires a sustained practice to master. Focus shifts over time, as seen in the following:

- In grades K-5, the focus is on the addition, subtraction, multiplication, and division of whole numbers; fractions and decimals; with a balance of concepts, skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.
- In middle school, multiplication and division develop into powerful forms of ratio and proportional reasoning. The properties of operations take on prominence as arithmetic matures into algebra. The theme of quantitative relationships also becomes explicit in grades 6-8, developing into the formal notion of a function by grade 8. Meanwhile, the foundations of high school deductive geometry are laid in

the middle grades. Finally, the gradual development of data representations in grades K-5 leads to statistics in middle school: the study of shape, center, and spread of data distributions; possible associations between two variables; and the use of sampling in making statistical decisions.
- In high school, algebra, functions, geometry, and statistics develop with an emphasis on modeling. Students continue to take a thinking approach to algebra, learning to see and make use of structure in algebraic expressions of growing complexity. As this description suggests, mathematical content in all grades is best approached in the ways envisioned by the Standards for Mathematical Practice.

The standards focus on crucial material so that students can have more time to discuss, reflect upon, and practice it. The standards treat mathematics as a coherent subject to promote the sense-making that fuels mastery. The principles of focus and coherence are the twin engines that must be carried forward in implementation efforts and substantiated in curricula and assessments.

Connecting Content and Practices

The Standards for Mathematical Content and Standards for Mathematical Practice are meant to be connected, as noted in the Common Core State Standards for Mathematics (page 8):

> *Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.*

The word *connect* in this passage is important. Separating the practices from the content is not helpful and is not what the standards require. The practices do not exist in isolation; the vehicle for engaging in the practices is mathematical content.

The Standards for Mathematical Practice should be embedded in classroom instruction, discussions, and activities. They describe the kind of mathematics teaching and learning to be fostered in the classroom. To promote such an environment, students should have opportunities to work on carefully designed standards-based mathematical tasks that can vary in difficulty, context, and type. Carefully designed standards-based mathematical tasks will reveal students? content knowledge and elicit evidence of mathematical practices. Mathematical tasks are an important opportunity to connect content and practices. To be consistent with the standards as a whole, assessment as well as curriculum and classroom activities must include a balance of mathematical tasks that provide opportunities for students to develop the kinds of expertise described in the practices.[1]

Higher Expectations: Conceptual Understanding, Fluency and Application

The standards are a rigorous set of expectations. According to these standards, it is not enough for students to learn procedures by rote. Nor, on the other hand, is it enough for students to ?understand the concepts? without being able to apply them to solve problems. Nor, finally, is it enough for students to learn the important procedures of mathematics without attaining skill and fluency in them.

- *Conceptual understanding:* A number of individual content standards use the word *understand*

in connection with important mathematical concepts. As the standards state (page 4),

There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a + b)(x + y) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding (a + b + c)(x + y).

Conceptual understanding will be assessed using both short tasks and performance-based tasks as part of PARCC?s commitment to measure the full range of the standards.

- *Procedural skill and fluency:* As the standards state (page 4), ?conceptual understanding and procedural skill are equally important.? Thus, at various grade levels, specific content standards use the word *fluently.* These standards will be assessed as part of PARCC?s commitment to measure the full range of the standards.

Wherever the word *fluently* appears in a content standard, the word means *quickly and accurately*. It means more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow: Fluent isn?t halting, stumbling, or reversing oneself. A key aspect of fluency in this sense is that it is not something that happens all at once in a single grade but requires attention to student understanding along the way. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

- *Application:* One of the mathematical practices is Modeling (MP.4), which sets an expectation that students will ?apply the mathematics they know to problems arising in everyday life, society, and the workplace.? Modeling is further developed as a conceptual category in high school, where it is explicitly linked to mathematical content standards using the star symbols (see pages 72 and 73 of *Common Core State Standards for Mathematics*). Furthermore, many individual content standards refer explicitly to real-world problems. The ability to apply mathematics will be assessed as part of PARCC?s commitment to measure the full range of the standards.

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[1] To align with strong instruction, PARCC assessments will include several types of tasks. The task types will allow for integration of the content and practice standards. Task types will include shorter items and longer, constructed items, which will vary in technical difficulty. While they are not part of this document, PARCC is currently developing prototype assessment tasks that will be made available in the near future.